MATH-805 Computational Mathematics

Credit Hours: 3-0

Prerequisite: None

Course Objectives: The main objective of this course is to train students to acquaint with the processes involved in numerical technique. The rigorous analysis of the numerical techniques to solve different problems pertaining to physical processes will be presented. Moreover, the students will get to know the programming sense of numerical procedures.

Core contents: Root finding techniques, Interpolation, Numerical differentiation, Runge-Kutta methods, Higher order method, Boundary value problem, Introduction to finite difference method for ODEs and PDEs

Course Contents: Newton's method for algebraic equations, interpolation and Lagrange polynomial, numerical differentiation, higher order Taylor methods, Runge-Kutta methods, higher-order differential equations and system of equations, the linear and nonlinear shooting methods, introduction to finite difference method for ODEs and PDEs

Course Outcomes: After reading this course one will be able to:

- Understand basics and advanced techniques in numerical methods
- Find solutions of system of nonlinear equations
- Solving IVP and BVP numerically
- Apply finite difference method to partial differential equations

Overview basics of numerical method algorithm and its implementation in software (MATLAB)

Textbook:

Numerical Methods for Engineers and Scientists Using MATLAB by Ramin S. Esfandiari, CRC Press, 2nd Edition, 2017

Computational Fluid Dynamics by Klasus A. Hoffmann and Steve T. Chiang, Fourth Edition, 2000.

Reference Books

Numerical Analysis By Richard L. Burden, J. Douglas Faires and Annette M. Burden, 10 E, Cengage Learning, 2016

Applied Numerical Analysis by Curtis F. Gerald and Patrick O. Wheatley, 7th Edition, Publisher: Pearson, 2003.

Theory and Application of Numerical Analysis by G. M. M. Phillips and Peter J. Taylor, 2nd edition, Academic Press, 1996

Numerical Analysis by David Kincaid and Ward Cheney, 7th Edition, Cengage Learning, 2012

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown		
Week	**Sections**	**Topic**
1	3.2, 3.3, 3.4, 4.7.2	• Review of Root Finding Methods Bisection method, Regula Falsi Method (Method of False Position), Fixed-Point Method • Fixed-Point Iteration Method for a System of Nonlinear Equations
2	3.5, 3.6, 4.7.1	• Review of Secant Method • Review of Newton's Method (Newton–Raphson Method), • Newton's Method for a System of Nonlinear Equations
3	4.3.4, 4.5, 4.6	• Tridiagonal Systems: Thomas Method • Iterative Solution of Linear Systems: Jacobi Iteration Method, Gauss–Seidel Iteration Method • ill-Conditioning and Error Analysis: Condition Number, Condition Number, Ill-Conditioning, Computational Error
4	5.5.1 5.5.3 5.5.5 5.6.1 5.6.2	Review of Interpolation Polynomial Interpolation: Lagrange Interpolating Polynomials, Newton Divided-Difference Interpolating Polynomials, Newton Forward-Difference Interpolating Polynomials, • Spline Interpolation Linear Splines, Quadratic Splines,

5	6.2.1 6.2.2 7.1 7.2	• Numerical Differentiation • Finite-Difference Formulas for Numerical Differentiation: Finite-Difference Formulas for the First Derivative and Second Derivative • Introduction to IVP • One-Step Methods
6	7.3	• Euler's Method: Error Analysis for Euler's Method Local And Global Truncation Errors Higher-Order Taylor Methods
7	7.4.1.1 7.4.1.2 7.4.1.3 7.4.1.4	• Runge–Kutta Method • Second-Order Runge–Kutta (RK2) Methods: Improved Euler's Method, Heun's Method Ralston's Method Graphical Representation of Heun's Method
8	7.4.2.1 7.4.2.2	• Third-Order Runge–Kutta (RK3) Methods: The Classical RK3 Method, Heun's RK3 Method
9	**Mid Semester Exam**	
10	7.4.3.1 7.4.3.2 7.4.5	• Fourth-Order Runge–Kutta (RK4) Methods: The Classical RK4 Method, Higher-Order Runge–Kutta Methods • Runge–Kutta Fehlberg (RKF) Method
11	7.6.2.1 7.6.2.2 7.6.2.3	• Numerical Solution of a System of First- Order ODEs Euler's Method for System, Heun's Method for System, Classical RK4 Method for Systems

12	7.7.1 7.7.2	• Stability: Euler’s Method Euler’s Implicit Method
13	8.4	• Shooting Method: Linear BVP Nonlinear BVP
14	8.5	• Finite-Difference Method: Finite-Difference Method, Linear BVP With Mixed BoundaryConditions
15	8.6 8.6.1	• MATLAB Built-In Function bvp4c for Boundary-Value Problems • Second-Order BVP
16	2.6 (Hofman n) CFD	• Finite difference Approximation for Mixed Partial Derivatives • Taylor series expansion
17	10.3.1 10.3.2	• Numerical Solution of Partial Differential Equations • Parabolic Partial Differential Equations • Crank–Nicolson (CN) Method
18	**End Semester Exam**	